Search for Dark Matter in $b \to s$ Transitions with Missing Energy

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Dedicated underground experiments searching for dark matter have little sensitivity to GeV and sub-GeV masses of dark matter particles. We show that the decay of B mesons to $K(K^*)$ and missing energy in the final state can be an efficient probe of dark matter models in this mass range. We analyze the minimal scalar dark matter model to show that the width of the decay mode with two dark matter scalars $B \to KSS$ may exceed the decay width in the Standard Model channel, $B \to K\nu\bar{\nu}$, by up to two orders of magnitude. Existing data from B physics experiments almost entirely exclude dark matter scalars with masses less than 1 GeV. Expected data from B factories will probe the range of dark matter masses up to 2 GeV.

INTRODUCTION

Although the existence of dark matter is firmly established through its gravitational interaction, the identity of dark matter remains a big mystery. Of special interest for particle physics are models of weakly interacting massive particles (WIMPs), which have a number of attractive features: well-understood mechanisms of ensuring the correct abundance through the annihilation at the freeze-out, milli-weak to weak strength of couplings to the "visible" sector of the Standard Model (SM), and as a consequence, distinct possibilities for WIMP detection. The main parameter governing the abundance today is WIMP annihilation cross section directly related to the dark matter abundance. In order to keep WIMP abundance equal or smaller than the observed dark matter energy density, the annihilation cross section has to satisfy the lower bound, $\sigma_{\rm ann}v_{\rm rel} \gtrsim 1$ pb, (see *e.g.* [1]). In all WIMP models studied to date the annihilation cross section is suppressed in the limit of very large or very small mass of a WIMP particle S. This confines the mass of a stable WIMP within a certain mass range, $m_{\rm min} \leq m_S \leq m_{\rm max}$, which we refer to as the Lee-Weinberg window [2]. This window is model-dependent and typcally extends from a few GeV to a few TeV. If the neutralino is the lightest stable supersymmetric particle, $m_{\rm min} \simeq 5$ GeV [3] but in other models of dark matter $m_{\rm min}$ can be lowered [4, 5].

Recently, WIMPs with masses in the GeV and sub-GeV range have been proposed as a solution to certain problems in astrophysics and cosmology. For example, sub-GeV WIMPs can produce a high yield of positrons in the products of WIMP annihilation near the centres of galaxies [6], which may account for 511 KeV photons observed recently in the emission from the Galactic bulge [7]. GeV-scale WIMPs are also preferred in models of self-interacting dark matter [8] that can rectify the problem with over-dense galactic centers predicted in numerical simulations with non-interacting cold dark matter.

Dedicated underground experiments have little sensitivity to dark matter in the GeV and sub-GeV range. Direct detection of the nuclear recoil from the scattering of such relatively light particles is very difficult because of the rather low energy transfer to nuclei, $\Delta E \sim v^2 m_S^2/m_{\rm Nucl} \lesssim 0.1$ KeV, which significantly weakens experimental bounds on scattering rates below m_S of few GeV, especially for heavy nuclei. Indirect detection via energetic neutrinos from the annihilation in the centre of the Sun/Earth is simply not possible in this mass range because of the absence of directionality. Therefore, the direct production of dark matter particles in particle physics experiments stands out as the most reliable way of detecting WIMPs in the GeV and sub-GeV mass range.

The purpose of this work is to prove that B decays can be an effective probe of dark matter near the lower edge of the Lee-Weinberg window. K decays can also be used for this purpose, but B decays have far greater reach, up to $m_S \sim 2.6$ GeV. In particular, we show that pair production of WIMPs in the decays $B \to K(K^*)SS$ can compete with the Standard Model mode $B \to K(K^*)\nu\bar{\nu}$. In what follows, we analyze in detail the "missing energy" processes in the model of the singlet scalar WIMPs [4, 9, 10] and use the existing data from B physics experiments to put important limits on the allowed mass range of scalar WIMPs.

The main advantage of the singlet scalar model of dark matter is its simplicity,

$$-\mathcal{L}_S = \frac{\lambda_S}{4}S^4 + \frac{m_0^2}{2}S^2 + \lambda S^2 H^\dagger H \qquad (1)$$
$$= \frac{\lambda_S}{4}S^4 + \frac{1}{2}(m_0^2 + \lambda v_{EW}^2)S^2 + \lambda v_{EW} S^2 h + \frac{\lambda}{2}S^2 h^2,$$

where H is the SM Higgs field doublet, $v_{EW} = 246$ GeV is the Higgs vacuum expectation value (vev) and h is the field corresponding to the physical Higgs, $H = (0, (v_{EW} + h)/\sqrt{2})$. It is important to recognize that the physical mass of the scalar S receives contributions from two terms, $m_S^2 = m_0^2 + \lambda v_{EW}^2$, and can be small, even if

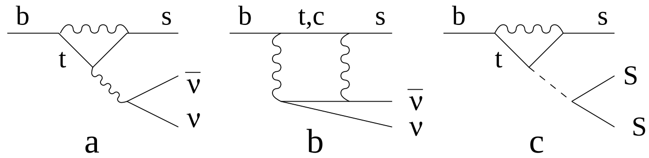

FIG. 1: Feynman diagrams which contribute to B meson decays with missing energy.

each term is on the order $\pm O(v_{EW}^2)$. Although admittedly fine-tuned, the possibility of low m_S is not *a priori* excluded and deserves further studies as it also leads to Higgs decays saturated by the invisible channel, $h \to SS$ and suppression of all observable modes of Higgs decay at hadronic colliders [4]. The minimal scalar model is not a unique possibility for light dark matter, which can be introduced more naturally in other models. If for example, the dark matter scalar S couples to the H_d Higgs doublet in the two-Higgs modification of (1), $\lambda S^2 H_d^\dagger H_d$, the fine-tuning can be relaxed if the ratio of the two electroweak vevs, $\tan\beta = \langle H_u\rangle/\langle H_d\rangle$ is a large parameter. A well-motivated case of $\tan\beta \sim 50$ corresponds to $\langle H_d\rangle \sim 5$ GeV, and only a modest degree of cancellation between m_0^2 and $\lambda\langle H_d\rangle^2$ would be required to bring m_S in the GeV range. More model-building possibilities open up if new particles, other than electroweak gauge bosons or Higgses, mediate the interaction between WIMPs and SM particles. If the mass scale of these new particles is smaller than the electroweak scale [5], sub-GeV WIMPs are possible without fine-tuning.

PAIR-PRODUCTION OF WIMPS IN B DECAYS

The Higgs mass m_h is heavy compared to m_S of interest, which means that in *all processes* such as annihilation, pair production, and elastic scattering of S particles, λ and m_h will enter in the same combination, $\lambda^2 m_h^{-4}$. In what follows, we calculate the pair-production of S particles in B decays in terms of two parameters, λ^2/m_h^4 and m_S, and relate them using the dark matter abundance calculation, thus obtaining the definitive prediction for the signal as a function of m_S alone.

At the quark level the decays of the B meson with missing energy correspond to the processes shown in Figure 1. The SM neutrino decay channel is shown in Figure 1a and 1b. The $b \to s$ Higgs penguin transition, Figure 1c, produces a pair of scalar WIMPs S in the final state, which likewise leave a missing energy signal. In this section, we show that this additional amplitude generates $b \to sSS$ decays that can successfully compete with the SM neutrino channel.

A loop-generated $b - s-$Higgs vertex at low momentum transfer can be easily calculated by differentiating the two-point $b \to s$ amplitude over v_{EW}. We find that to leading order the $b \to sh$ transition is given by an effective interaction (see *e.g.,* [11])

$$\mathcal{L}_{bsh} = \left(\frac{3g_W^2 m_b m_t^2 V_{ts}^* V_{tb}}{64\pi^2 M_W^2 v_{EW}}\right)\bar{s}_L b_R h + (h.c.). \quad (2)$$

Using this vertex, Eq. (1) and safely assuming $m_h \gg m_b$, we integrate out the massive Higgs boson to obtain the effective Lagrangian for the $b \to s$ transition with missing energy in the final state:

$$\mathcal{L}_{b\to s\not{E}} = \frac{1}{2}C_{DM} m_b \bar{s}_L b_R S^2 - C_\nu \bar{s}_L \gamma_\mu b_L \bar{\nu}\gamma_\mu \nu + (h.c.). \quad (3)$$

Leading order Wilson coefficients for the transitions with dark matter scalars or neutrinos in the final state are given by

$$C_{DM} = \frac{\lambda}{m_h^2}\,\frac{3g_W^2 V_{ts}^* V_{tb}}{32\pi^2}\,x_t \quad (4)$$

$$C_\nu = \frac{g_W^2}{M_W^2}\,\frac{g_W^2 V_{ts}^* V_{tb}}{16\pi^2}\left[\frac{x_t^2+2x_t}{8(x_t-1)} + \frac{3x_t^2-6x_t}{8(x_t-1)^2}\ln x_t\right],$$

where $x_t = m_t^2/M_W^2$.

We would like to remark at this point that the numerical value of C_{DM} is a factor of few larger than C_ν,

$$\frac{C_{DM}}{C_\nu} \simeq \frac{4.4\lambda M_W^2}{g_W^2 m_h^2}, \quad (5)$$

if $\lambda m_h^{-2} \sim O(g_W^2 M_W^{-2})$. This happens despite the fact that the effective bsh vertex is suppressed relative to bsZ vertex by a small Yukawa coupling $\sim m_b/v_{EW}$. The $1/v_{EW}$ in (2) is compensated by a large coupling of h to S^2, proportional to λv_{EW}, and m_b is absorbed into the definition of the dimension 6 operator $m_b\bar{s}_L b_R S^2$.

We concentrate on exclusive decay modes with missing energy, as these are experimentally more promising than inclusive decays and give sensitivity to a large range of m_S. A limit on the branching ratio has recently been reported by BaBar collaboration, $\mathrm{Br}_{B^+\to K^+\nu\bar{\nu}} < 7.0\times10^{-5}$ at 90% c.l. [12], which improves on a previous CLEO limit [13], but is still far from the SM prediction $\mathrm{Br}(B \to K\nu\bar{\nu}) \simeq (3-5)\times10^{-6}$ (See, e.g. [14]). We use the result for $\mathcal{L}_{b\to s\not{E}}$ along with the hadronic form factors determined via light-cone sum rule analysis in [15] and related to the scalar $B \to K$ transition in [16], to produce the amplitude of $B \to KSS$ decay,

$$\mathcal{M}_{B\to KSS} = C_{DM} m_b \frac{M_B^2 - M_K^2}{m_b - m_s} f_0(q^2), \quad (6)$$

where $q^2 = \hat{s} = (p_B - p_K)^2$ and the form factor for $B \to K$ transition is approximated as $f_0 \simeq 0.3\exp\{0.63\hat{s}M_B^{-2} - 0.095\hat{s}^2 M_B^{-4} + 0.591\hat{s}^3 M_B^{-6}\}$.

The differential decay width to a K meson and a pair of WIMPs is given by

$$\frac{d\Gamma_{B^+\to K^+SS}}{d\hat{s}} = \frac{x_t^2 C_{DM}^2 f_0(\hat{s})^2}{512\pi^3}\,\frac{I(\hat{s},m_S)m_b^2(M_B^2-M_K^2)^2}{M_B^3(m_b-m_s)^2}, \quad (7)$$

where $I(\hat{s}, m_S)$ reflects the available phase space,

$$I(\hat{s}, m_S) = [\hat{s}^2 - 2\hat{s}(M_B^2 + M_K^2) + (M_B^2 - M_K^2)^2]^{\frac{1}{2}} [1 - 4m_S^2/\hat{s}]^{\frac{1}{2}}.$$

From Eq. (7) and the prediction for the SM neutrino channel, we obtain the total branching ratio for the B^+ to K^+ decay with missing energy in the final state,

$$\begin{aligned} \mathrm{Br}_{B^+\to K^+ + \not{E}} &= \mathrm{Br}_{B^+\to K^+\nu\bar{\nu}} + \mathrm{Br}_{B^+\to K^+ SS} \\ &\simeq 4\times 10^{-6} + 2.8\times 10^{-4}\kappa^2 F(m_S). \end{aligned} \quad (8)$$

Eq. (8) uses the parametrization of $\lambda^2 m_h^{-4}$,

$$\kappa^2 \equiv \lambda^2 \left(\frac{100\ \mathrm{GeV}}{m_h}\right)^4, \quad (9)$$

and the available phase space as a function of the unknown m_S,

$$F(m_S) = \int_{\hat{s}_{min}}^{\hat{s}_{max}} f_0(\hat{s})^2 I(\hat{s}, m_S)\, d\hat{s} \left[\int_{\hat{s}_{min}}^{\hat{s}_{max}} f_0(\hat{s})^2 I(\hat{s}, 0)\ d\hat{s}\ \right]^{-1}$$

Notice that $F(0) = 1$ and $F(m_S) = 0$ for $m_S > \frac{1}{2}(m_B - m_K)$ by construction. Similar calculations can be used for the decay $B \to K^* SS$,

$$\mathrm{Br}_{B^+\to K^{+*} + \not{E}} \simeq 1.3\times 10^{-5} + 4.3\times 10^{-4}\kappa^2 F(m_S). \quad (10)$$

with an analogous form factor.

For light scalars, $m_S \sim$ few 100 MeV, and $\kappa \sim O(1)$ the decay rates with emission of dark matter particles are ~ 50 times larger than the decay with neutrinos in the final state! This is partly due to a larger amplitude, Eq. (5), and partly due to phase space integral that is a factor of a few larger for scalars than for neutrinos if m_S is small.

ABUNDANCE CALCULATION AND COMPARISON WITH EXPERIMENT

The scalar coupling constant λ and the scalar mass m_S are constrained by the observed abundance of dark matter. For low m_S, as shown in [4], the acceptable value of κ is $\kappa \sim O(1)$. Here we refine the abundance calculation for the range $0 < m_S < 2.4$ GeV in order to obtain a more accurate quantitative prediction for κ. The main parameter that governs the energy density of WIMP particles today, which we take to be equal to the observed value of $\Omega_{DM} h^2 \sim 0.13$ [17], is the average of their annihilation cross section at the time of freeze-out. This cross section multiplied by the relative velocity of the annihilating WIMPs is fixed by Ω_{DM} and can be conveniently expressed [4] as

$$\sigma_{\mathrm{ann}}\, v_{rel} \;=\; \frac{8 v_{EW}^2 \lambda^2}{m_h^4} \times \left(\lim_{m_{\tilde{h}}\to 2m_S} m_{\tilde{h}}^{-1} \Gamma_{\tilde{h}X}\right) \quad (11)$$

Here $\Gamma_{\tilde{h}X}$ denotes the partial rate for the decay, $\tilde{h} \to X$, for a virtual Higgs, $\tilde{h}$, with the mass of $m_{\tilde{h}} = 2E_S \simeq 2m_S$. Notice that Eq. (11) contains the same combination $\lambda^2 m_h^{-4}$ as (8). The zero-temperature width $\Gamma_{\tilde{h}X}$ was extensively studied two decades ago in conjunction with searches for light Higgs [18, 19, 20].

For m_S larger than m_π the annihilation to hadrons dominates the cross section, which is therefore prone to considerable uncertainties. At a given value of m_S, we can predict $\Gamma_{\tilde{h}X}$ within a certain range that reflects these uncertainties. With the use of (11), this prediction translates into the upper (A) and the lower (B) bounds on $\kappa(m_S)$, which we insert into Eq. (8) and plot the resulting $\mathrm{Br}_{B^+\to K^+ + \not{E}}$ in Figure 2.

In the interval 150 MeV $\leq m_S \lesssim$ 350 MeV the annihilation cross section is dominated by continuum pions in the final state, and can be calculated with the use of low-energy theorems [18] to good accuracy. Requiring $\kappa^2 < 4\pi$ allows to determine the lower end of the Lee-Weinberg window in our model, $m_{\min} \sim 350$ MeV. In the interval 350 MeV$-$650 MeV the strangeness threshold opens up, and annihilation into pions via the s-channel f_0 resonance become important. The strength of this resonance and its width and position at freeze-out temperatures, $T \sim (0.05 - 0.1)m_S$, are uncertain. Curve B in this domain of Figure 2 assumes the f_0 resonance is completely insensitive to thermal effects and has the *minimum* width quoted by PDG [21] which maximizes $\Gamma_{\tilde{h}X}$, whereas curve A corresponds to a complete smearing of the f_0 resonance by thermal effects and much lower value of $\Gamma_{\tilde{h}X}$. Above m_S =1 GeV, curve B takes into account the annihilation into hadrons mediated by $\alpha_s(G_{\mu\nu})^2$ *with* the two-fold enhancement suggested by charmonium decays [18] whereas curve A uses the perturbative formula. The charm threshold is treated simply by the inclusion of open charm quark production at a low threshold ($m_c \simeq 1.2$ GeV) in B curve and at a high threshold ($m_S > m_D$) in curve A. Both curves include the τ threshold. There are no tractable ways of calculating the cross section in the intermediate region 650 MeV $\lesssim m_S \lesssim$ 1 GeV. However, there are no particular reasons to believe that the annihilation into hadrons will be significantly enhanced or suppressed relative to the levels in adjacent domains. In this region, we interpolate between high- and low-energy sections of curves A and B. Thus, the parameter space consistent with the required cosmological abundance of S scalars calculated with generous assumptions about strong interaction uncertainties is given by the area between the two curves, A and B.

Figure 2 presents the predicted range of $\mathrm{Br}_{B^+\to K^+ + \not{E}}$ as a function of m_S and is the main result of our paper. The SM "background" from $B \to K\nu\bar{\nu}$ decay is subdominant everywhere except for the highest kinematically allowed domain of m_S. To compare with experimental results [12, 13], we must convert the limit on $\mathrm{Br}_{B^+\to K^+\nu\bar{\nu}}$

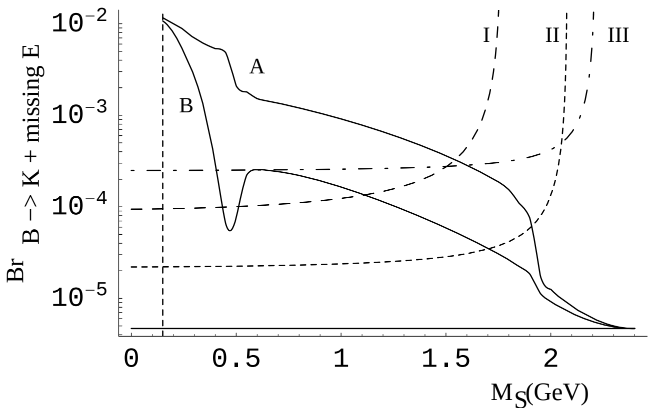

FIG. 2: Predicted branching ratios for the decay $B \to K+$ *missing energy*, with current limits from BaBar (I) [12], CLEO (III) [13] and expected results from BaBar (II). Parameter space above curves I and III is excluded. The horizontal line shows the SM $B \to K\nu\bar{\nu}$ signal. Parameter space to the left of the vertical dashed line is also excluded by $K^+ \to \pi^+ \not{E}$.

to a more appropriate bound on $\mathrm{Br}_{B^+\to K^+ + \not{E}}$ according to the following procedure. We multiply the experimental limit of 7.0×10^{-5} by a ratio of two phase space integrals, $F(m_S, \hat{s}_{min})/F(m_S, \hat{s}_{exp})$, where s_{exp} is determined by the minimum Kaon momentum considered in the experimental search, namely 1.5 GeV. This produces an exclusion curve, nearly parallel to the m_S axis at low m_S, and almost vertical near the experimental kinematic cutoff. The current BaBar results (curve I) exclude $m_S < 430$ MeV, as well as the region 510 MeV$< m_S < 1.1$ GeV, and probe the allowed parameter space for dark matter up to $m_S \sim 1.5$ GeV. Generalized model with N component dark matter scalar gives N^2-fold increase in the branching ratio [4], and thus greater sensitivity to m_S.

The B factories will soon have larger data samples and can extend the search to lower Kaon momenta. The level of sensitivity expected from an integrated luminosity $\mathcal{L}$ of 250 fb^{-1} and momentum cutoff of 1 GeV is shown by curve II, which assumes that the sensitivity scales as $\mathcal{L}^{-1/2}$, as suggested by the analysis in [12]. In reality, the experimental limit will extend to Kaon momenta below 1 GeV where the sensitivity will gradually degrade due to increasing backgrounds; however, we expect the implication of curve II to remain valid, namely that the B factories will probe scalar dark matter up to 2 GeV.

If $m_S \lesssim 150$ MeV, the decay $K^+ \to \pi^+ SS$ becomes possible. The width for this decay can be easily calculated in a similar fashion to $b \to s$ transition. The concordance of the observed number of events with the SM prediction [22] rules out scalars in our model with $m_S < 150$ MeV. This exclusion limit is shown by a vertical line in Figure 3. It is below $m_{\min}$ of 350 MeV.

To conclude, we have demonstrated that the $b \to s$ transitions with missing energy in the final state can be an efficient probe of dark matter when pair production of WIMPs in B meson decays is kinematically allowed. In particular, we have shown that the minimal scalar model of dark matter with the interaction mediated by the Higgs particle predicts observable rates for $B^+ \to K^+$ and missing energy. A large portion of the parameter space with $m_S \lesssim 1$ GeV is already excluded by current BaBar limits. New experimental data should probe a wider range of masses, up to $m_S \sim 2$ GeV. The limits obtained in this paper have important implications for Higgs searches, as the existence of relatively light scalar WIMPs leads to the Higgs decays saturated by invisible channel. Given the astrophysical motivations for GeV and sub-GeV WIMPs combined with insensitivity of dedicated dark matter searches in this mass range, it is important to extend the analysis of $b \to s$ transition with missing energy onto other models of light dark matter.

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